

**#34 PRACTICE TEST 3**

[3.7 -- 4.6(I)]

Give exact answers in simplest form unless told to do otherwise. Points are awarded based on your use of calculus.

1. Consider the curve given by the function  $\ln(x^2) - 3y = 2 - 3x\ln(y)$ .

(a) Find  $\frac{dy}{dx}$ . SHOW ALL WORK. SIMPLIFY ANSWER AS MUCH AS POSSIBLE — DO NOT LEAVE YOUR ANSWER AS A COMPLEX FRACTION OR WITH NEGATIVE EXPONENTS. [12 POINTS]

(b) Find the equation of the tangent line to this curve at  $(e, e)$ . SHOW ALL WORK AND GIVE SIMPLEST, EXACT ANSWER. [5 POINTS]

2. Find the local linearization of  $\frac{2}{1-5x^2}$  near  $x = 1$ . SHOW ALL WORK. [16 POINTS]

3. The function  $y = at + bt^{-1}$  has a local minimum at the point  $(6, 3)$ . Find the values of  $a$  and  $b$ . SHOW ALL WORK. [17 POINTS]

4. A pigeon is released from a canoe (point  $C$ ) floating on a lake. Because of falling air over the cool water, the energy required to fly one meter over the lake is twice the corresponding energy  $e$  required for flying over the bank ( $e = 3$  joules/meter). To minimize the energy required to fly from  $C$  to the nest,  $N$ , the pigeon heads to a point  $T$  on the bank and then flies along the bank to  $N$ . The distance  $\overline{PN}$  is 4000 meters, and  $\overline{CP}$  is 500 meters. Point  $P$  is a point directly across the lake from  $C$  so that  $\overline{CP}$  is perpendicular to  $\overline{PN}$ . Find the distance  $\overline{PT}$  that will minimize energy. GIVE THE EXACT ANSWER AND THE APPROXIMATE ANSWER ROUNDED TO THE NEAREST METER. SHOW ALL WORK. [17 POINTS]

5. Three cheerful problems

- (a) Find the points of inflection for this function,  $g(z) = z^4 + 2z^3 - 36z^2 + 60z + 17$ . Be sure to establish that your candidates for POI are indeed inflection points. SHOW ALL WORK. [8 POINTS]

- (b) Let  $h(t) = \frac{1}{\sqrt{\cosh(t)}}$ . Find  $h'(t)$ . [7 POINTS]

- (c) Let  $f(A) = \ln(A^2)$  on the interval  $[1, 3]$ . Find the coordinates,  $(A, f(A))$ , of the global minimum and the global maximum of  $f$ . SHOW ALL WORK AND GIVE SIMPLEST, EXACT ANSWERS.. [8 POINTS]

6. The gravitational force,  $F$  (in *newtons*), on a rocket at a distance,  $r$  (in *km*), from the center of the earth is given by

$$F = \frac{k}{r^2}$$

where  $k = 10^{13}$  *newton · km<sup>2</sup>*. When the rocket is  $10^4$  *km* from the center of the earth, it is moving away at  $0.2$  *km/sec*. How fast is the gravitational force changing at that moment?

[10 POINTS]